ON THE TRANSFER OF ENERGY IN LAYERS OF FUR

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INTRODUCTION

The interaction between an animal and its energy environment is of general interest in biological studies. The amount of energy exchanged may be determined on a long term basis from metabolic studies. The intent of thermal modeling (e.g., Birkebak, 1966; Gates, 1962) is to provide a mathematical model of both animal and energy environment and thereby allow the researcher to calculate the energy exchange at a given instant. We have undertaken the investigation of one aspect of thermal modeling, the theoretical prediction of the thermal conductivity of fur.

The transfer of energy occurs by conduction, convection, radiation, and evaporation. In cases where effects of evaporation are confined to a region very near the skin of a fur bearing animal, the processes of conduction, convection, and radiation within fur are not coupled to the evaporative energy exchange and can be considered separately. Then the flux of energy from the skin of an animal is given by

$$q_f = k_{eff}\,(\Delta T/L), \tag{1}$$

where q_f is the rate of energy transfer per unit area of skin (watts per square meter), ΔT is the temperature difference across the fur layer (kelvin), L is the thickness of the fur layer (meters), and k_{eff} is an effective thermal conductivity (watts per meter-kelvin) of fur and accounts for conduction, convection, and radiation within the fur. Effects of the external air and the penetration of direct solar radiation are not included (see the subsequent discussion of convection and direct solar radiation).

The techniques reported herein permit the calculation of the value of k_{eff} applying to the fur covering a section of an animal's body when certain properties of the fur are known. It is thus necessary to determine values of k_{eff} for regions such as the sides, stomach, back, legs, neck, and head in order to meaningfully apply the thermal modeling equations.

STRUCTURE OF FUR

The arrangement, size, and number of hairs and the thickness of the fur are a dominating influence on the transfer of energy by radiation and convection and by conduction along the hairs and through the air in the fur. A geometrical model of the structure of fur is necessary in the derivation of the equations describing energy transfer.

The coordinate system applying to the particular fur model chosen is shown in Fig. 1 *A*. The x direction is parallel to the skin and aligned with the direction of the grain of the fur while the y direction is normal to the skin. The z direction is normal to both x and y such that x, y, and z form a right-handed coordinate system. The angle between the axis of a hair and a normal to the skin is, at a given value of y, θ_f and ϕ' is the angle between the projection of a hair onto the skin and the grain (x) direction. The coordinates θ and ϕ are introduced for use in the energy equations.

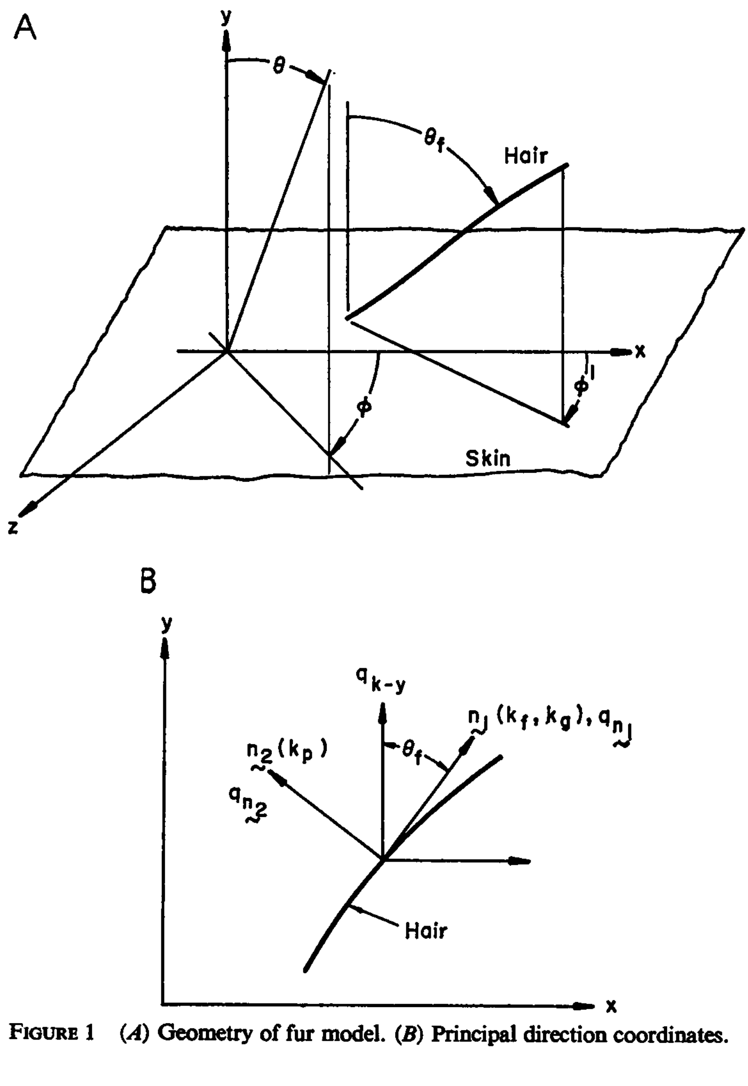

FIGURE 1 (*A*) Geometry of fur model. (*B*) Principal direction coordinates.

At each value of y, it is assumed that all hairs crossing an infinitesimal surface element parallel to the skin are characterized by one value of θ_f.

The equations for the conduction and radiation energy fluxes require knowledge of the fraction of area (parallel to the skin) and the fraction of the fur volume occupied by hair at a given distance from the skin. For the fur model depicted in Fig. 1, this quantity is given by (Davis, 1972)

$$\rho_{eff}/\rho_f = n_f(\pi \overline{d_f^2}/4)/\cos\theta_f, \tag{2}$$

where ρ_{eff} is an effective mass density of the fur (kilograms per cubic meter), ρ_f is the mass density of the hair (kilograms per cubic meter), n_f is the number of hairs per unit area of skin (meter^{-2}), and $\overline{d_f^2}$ is the average of the squares of the hair diameters (square meters). The effective mass density, ρ_{eff}, is defined as the ratio of the mass of the hair contained in a small volume element surrounding the area of interest to the volume of the element. The area and volume fractions, equal to ρ_{eff}/ρ_f, are thus given on a local basis by Eq. 2. The value will change both because any of the fur properties (n_f, d_f, θ_f, ρ_f) vary over the surface of the animal and because θ_f and d_f are functions of y even in a particular region. Note that piloerection decreases the value of θ_f and thus decreases ρ_{eff}/ρ_f.

As can be seen from Fig. 1, fur is an anisotropic material; there is a strong dependence of the transfer of quantities such as energy on the direction in which the transfer takes place. A set of coordinates called principal directions can be found for an anisotropic material. Their significance lies in the fact that (energy) transfer in one of these principal directions depends only on a (temperature) gradient in that same direction. The three principal directions of the fur model shown in Fig. 1 are (*a*) aligned with the axis of the hair, (*b*) normal to the hair axis and lie in the $x - y$ plane, (*c*) coincident with the z direction. The first two of these can vary as the value of θ_f varies, i.e. with distance from the skin.

It can thus be expected that there will be energy transfer in both the x and y direction if there are temperature gradients in either the x or y directions. Energy transfer along the z axis occurs only if there is a z direction temperature gradient.[1]

ENERGY TRANSFER

The interaction of convection, conduction, and radiation occurring within fur results in a complex energy transfer process. In this section, we present a method for calculating the effective thermal conductivity in the absence of convection within the

[1] In the treatment of radiation, we find it necessary to account for the fact that all hairs do not lie in the $x - y$ plane but are displaced by the azimuthal angle ϕ' as shown in Fig. 1. If, in the treatment of conduction, we were to consider individual hairs, then it follows that the z direction would no longer be a principle coordinate. However, since we are dealing with a large number of hairs, then by definition of the grain direction, the sums of both the individual values of ϕ' and the z direction energy fluxes (arising from temperature gradients in either of the x or y directions) are identically zero.

fur and then describe a technique which allows convection to be taken into account utilizing an empirical correction and the previously calculated value of k_{eff}.

In the absence of convection, the temperature gradient in the y direction, $\partial T/\partial y$ (kelvin per meter) is much larger than that in either of the x and z directions. It is assumed that $\partial T/\partial x$ and $\partial T/\partial z$ are negligible and it is at most necessary to find the energy fluxes associated with $\partial T/\partial y$ (energy flux terms arising from $\partial T/\partial x$ and $\partial T/\partial z$ are given by Davis, 1972). This task can be further simplified. Since there is no z direction temperature gradient (and z is a principle direction), there is no energy transfer in the z direction. There will be energy transfer terms in the x and y directions, denoted for the present by q_x and q_y (watts per square meter), due to $\partial T/\partial y$. The flux of energy from the surface is a vector quantity, q, whose magnitude is

$$q = \sqrt{q_x^2 + q_y^2},$$

and which is inclined at an angle γ to the normal to the skin (the vector q is in the $x - y$ plane)

$$\cos\gamma = q_y/q.$$

The amount of energy leaving the skin per unit time, Q (watts) is equal to the product of the projection of the vector q onto the normal to the skin and of the area of the skin

$$Q = (q\cos\gamma)\,A,$$

where A is the area of the skin. Since q_f, as given by Eq. 1, is by definition equal to Q/A it follows that

$$q_f = q\cos\gamma = q_y\,.$$

Therefore, it is only necessary to find the radiation and conduction energy fluxes in the y direction in order to calculate k_{eff}.

Conduction

Energy is conducted through a fur layer along hairs, through the air pervading the layer, across hairs, and between hairs at points where they are in contact with one another. The thermal conductivity of fur is thus directionally dependent.

Fig. 1 *B* is a schematic representation of an $x - y$ plane through a fur layer. At a particular value of y, the two principle directions, respectively, are parallel to and normal to the axes of the hairs. The two values of thermal conductivity applicable to the former are that of hair, k_f, and that of air, k_a. When the fraction of area occupied by hair is known, then the energy flux can be calculated. Values of k_f must be obtained experimentally. A method for finding this thermal conductivity for larger hairs (of diameter greater than approximately 40 μm) is described by

Davis (1972). Typical values of k_f for hairs from tanned pelts are approximately 10 times that of air. The thermal conductivity k_p is more difficult to find. It includes effects of hair contracts, conduction across hairs, and through the air. The value of k_p is estimated by using a two-dimensional flux plot (see discussion by Davis, 1972). This assumes that the thermal resistance at points of hair contact is much larger than either that across the hair or the air, and that k_f is approximately 10 times larger than k_g.

The energy flux in the y direction is obtained by vector addition of the fluxes in the $\mathbf{n}_1$ and $\mathbf{n}_2$ directions. The magnitude of this flux is

$$q_{k-y} = q_{n_1} \cos \theta_f + q_{n_2} \sin \theta_f .$$

In terms of $\partial T/\partial y$,

$$q_{n_1} = -\{(\rho_{eff}/\rho_f)\, k_f + (1 - [\rho_{eff}/\rho_f])\, k_g\} \cos \theta_f(\partial T/\partial y)$$

$$q_{n_2} = -k_p \sin \theta_f(\partial T/\partial y),$$

where the first term in curly brackets in the equation for q_{n_1} is the flux of energy through the hairs and the second term is the energy flux through the air.

Thus, if we define the thermal conductivity in the y direction as k_y

$$q_{k-y} = -k_y(\partial T/\partial y), \tag{3}$$

it follows that

$$k_y = \{(\rho_{eff}/\rho_f)k_f + (1 - [\rho_{eff}/\rho_f])k_g\}\cos^2 \theta_f + k_p \sin^2 \theta_f . \tag{4}$$

For convenience, we define

$$k_1 = (\rho_{eff}/\rho_f)k_f + (1 - [\rho_{eff}/\rho_f])k_g , \tag{4a}$$

so that

$$k_y = k_1 \cos^2 \theta_f + k_p \sin^2 \theta_f . \tag{4b}$$

The effect of fur structure on conduction energy transfer is apparent in Eq. 4. Conduction is directly proportional to the fraction of the fur occupied by hair. As ρ_{eff}/ρ_f becomes small, the number of hair contacts and the importance of conduction through or along hairs decreases. The transverse conductivity k_p then approaches k_g for fur with ρ_{eff}/ρ_f less than, approximately, 0.05, and $k_y \simeq k_g$ to within about 10%. For heavier furs where ρ_{eff}/ρ_f is about 0.15, it is necessary to obtain values of k_f and k_p in order to find the anisotropic thermal conductivities.

Radiation

Development of equations describing radiative energy transfer in an emitting-absorbing material, such as fur, is accomplished in two distinct phases. The first is concerned with mathematically modeling the material to obtain the extinction coefficient, a quantity which accounts for the absorption and scattering (reflection) of radiant energy within the material. The second phase is the specification of the actual energy transfer terms.

Radiant energy transfer in fur arises from emission by the skin and the hairs at a rate depending on the local temperature and from penetration of the fur by external radiation fields. This latter energy is of two types: direct solar radiation and diffuse radiation (including solar radiation scattered in the atmosphere, i.e. skylight, and infrared radiation from the surrounds).

Hairs may absorb, transmit, or scatter incident radiant energy. Because it is not presently possible to predict the radiation properties of hair, experimental values for the emissivity, transmissivity, and reflectivity in the appropriate spectral region are used. This is necessary since the spectral radiation properties depend upon the hair diameters. Hagar and Steere (1967) have pointed out both this and that the hairs (fibers in their work) are not always opaque to thermal radiation. This particularly applies to thermal (infrared) radiation emitted by the skin and the hairs. With the restriction that radiation properties are obtained experimentally, the skin and the hair are assumed to emit diffusely. The emissivity of the skin is assumed to be the unity.

The model of fur structure presented previously must be extended to account for radiation. The distribution of hairs in the ϕ' direction is of negligible importance for modes of energy transfer other than radiation. However, failure to account for this distribution results in a singularity in the radiation equations. The physical interpretation of this singularity is that it is always possible to view the fur in a particular direction ($\theta = \theta_f$, $\phi = 0$) and see the skin. For most animals, this is not possible. For this reason, we add an additional function to the fur model. This function is to describe what an observer would see if he looked along the grain direction (x) from a point removed from the fur but in the $x - y$ plane (see Fig. 1 *A*). By definition of the grain direction he would note that as many hairs crossed the $x - y$ plane from the right as from the left. The distribution function must therefore by symmetric about the grain. In addition, he would note that the value of the azimuthal angle ϕ' (Fig. 1 *A*) is small for all hairs so the distribution function must decay rapidly. We assume that there is a maximum value of ϕ', called ϕ_f such that the magnitude of ϕ' is less than ϕ_f for all hairs.

The following distribution function is chosen because it meets the above criterion and for its mathematical simplicity

$$\frac{1}{n_f}\frac{dn_f'}{d\phi'} = \left[\cos^2\left(\frac{\pi}{2}\frac{\phi'}{\phi_f}\right)\right] \Big/ \phi_f, \qquad |\phi'| \leqq \phi_f. \tag{5}$$

At any value of $y(0 \leqq y \leqq L)$, n_f' is the number of hairs per unit area with the angle ϕ' between the grain direction and the projection of the hair axis onto the skin.

The derivation of a function describing the average distance traveled in some specified direction θ, ϕ (see Fig. 1 *A*) by a packet of radiant energy—a photon—proceeds from the definition of the probability P that a photon will sucessfully cross a volume element in the fur

$$P = e^{-\delta s\beta/\lambda}, \tag{6}$$

where δs is the path length, β is a form of an extinction coefficient (the usual definition is β/λ), and λ is the photon mean free path. The probability of capture of a photon within the volume element, $1 - P$, is equal to the ratio of the area projected by the hairs in the θ, ϕ direction to that projected by the volume element in the same direction.

Eq. 5 is used in the calculation of the area projected by the hairs in a given direction (see Davis, 1972). The photon mean free path is then given by

$$1/\lambda = (n_f d_f'/\cos\theta_f)F \tag{7}$$

where d_f' is the average hair diameter and

$$F = \left\{1 - \left[\sin\theta\cos\phi\sin\theta_f\frac{\sin\phi_f}{\phi_f}\frac{\pi^2}{\pi^2 - \phi_f^2} + \cos\theta\cos\theta_f\right]^2\right\}^{1/2}. \tag{8}$$

The function F has the property that it does not vanish for any value of θ and ϕ if θ_f and ϕ_f are not zero and so the photon mean free path remains finite for all viewing angles.

That photon mean free path, as given in Eq. 7 is a function only of the structure of the fur. The particular form of the structure is fixed by Eqs. 2 and 5 and results in the function F being given by Eq. 8. From these equations it can be seen that variation of θ_f with y is built into the model. Further, Eq. 8 is only weakly sensitive to the value of ϕ_f and so a value of $\phi_f = 10°$ is used (Davis [1972] gives value of the radiation functions for $\phi_f = 10°$ and 20°. There is approximately a 2% difference).

With the function F known, the radiative energy transfer is calculated by fixing attention on a volume element in the fur. The amount of radiant energy passing through this volume element in a given θ, ϕ direction from all sources (skin, external radiation fields, other volume elements in the fur) is determined and the expression which results is then integrated over the fur volume. The technique used here is called the two flux model of radiative transfer and is described in detail by Sparrow and Cess (1966).

Since conduction is the dominant energy transfer process in fur, the temperature terms in the radiation equations can be simplified to give the following for the transfer of diffuse radiation in fur (Davis, 1972).

$$q_{r-y} = (2\sigma/\pi)(T_a^4 - T^4)\int_0^{\pi}\int_0^{\pi/2} e^{-\eta N_f F/\cos\theta}\cos\theta\sin\theta\, d\theta\, d\phi$$

$$+ (2\sigma/\pi)(T^4 - T^4_{\infty,r}) \int_0^{\pi}\int_0^{\pi/2} e^{-(1-\eta)N_f F/\cos\theta} \cos\theta \sin\theta \, d\theta \, d\phi$$

$$- (8\sigma T^3/N_f \pi) L(\partial T/\partial y) \int_0^{\pi}\int_0^{\pi/2} f' \cos^2\theta \sin\theta \, d\theta \, d\phi, \quad (9)$$

where

$$N_f = (n_f d_f \beta L/\cos\theta_f) = 4(\rho_{eff}/\rho_f)\, \beta\, L/\pi d_f\,, \quad (10)$$

and

$$f' = \frac{L}{N_f}\left\{\left[\frac{2 - \exp(-\eta N_f F/\cos\theta) - \exp(-[1-\eta]N_f/F\cos\theta)}{F}\right]\right.$$

$$\left. - N_f\left[\frac{(1-\eta)}{\cos\theta}\exp(-[1-\eta]N_f F/\cos\theta) + \frac{\eta}{\cos\theta}\exp(-N_f\eta F/\cos\theta)\right]\right\}. \quad (11)$$

In these equations, η is a dimensionless distance from the skin, $\eta = y/L$, σ is the Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4, T_a is the local skin temperature, $T_{\infty,r}$ is the equivalent blackbody radiation temperature of the external diffuse radiation field, and q_{r-y} is the flux of diffuse radiant energy in the y direction.

The dimensionless quantity N_f is a measure of the optical thickness of the fur. It is of the order of magnitude of the ratio of the fur thickness and the mean free path λ. Thus, for values of N_f greater than, approximately, 10, the fur may be said to be optically thick. Values of N_f for tanned furs (Davis, 1972) range from 17 for rabbit fur to 32 for deer fur.

The extinction coefficient is a function of the absorption and scattering coefficients. For optically thick materials this function is simply a sum of these two quantities (Sparrow and Cess, 1966). Thus, for fur

$$\beta = \alpha + \rho, \quad (12)$$

where α and ρ, respectively, are the absorptivity and reflectivity of hair. It should be emphasized that this is not an a priori approximation but is derived from the general equations of radiative transfer for absorbing, emitting, and scattering media when the optically thick condition is imposed. Additional terms involving the absorption and scattering coefficients appear in the equations of radiative transfer if the material is not optically thick (or optically thin, i.e. $N_f \ll 1$). These are of negligible importance for calculating radiative transfer in fur.

In Eq. 9, the first term is the net radiation from the skin through a plane located at a distance y above the skin. If we set $F = 0$ in this term, it reduces to $\sigma(T_a^4 - T^4)$, the net energy exchange between two black planes. The double integral, called F_1, accounts for the decrease in the energy exchange between these two planes because of the presence of the hairs. The decay of F_1 with increasing values of N_f is very rapid. Values of F_1 for $\theta_f = 30°$, and 45°, $\phi_f = 10°$, at $\eta = 0.5$ are given in Table I.

TABLE I

VALUES OF THE RADIATION FUNCTION F_1 AT $\eta = 0.5$ AND $\phi_f = 10°$

	$10^4 F_1$ (η = 0.5)	
N_f	θ_f = 30°	θ_f = 45°
10	693.3	463.6
20	163.5	80.92
30	54.44	21.43
40	20.46	6.642
50	8.156	2.226
60	3.371	0.781
70	1.426	0.282
80	0.613	0.104
90	0.267	0.038
100	0.177	0.014

The second term is similar to the first and gives the net radiant transfer passing through the plane at y toward the external radiation sink(s). It decays in exactly the same fashion as the first except that its maximum value occurs at the outer edge of the fur layer rather than at the skin.

The third term in Eq. 9, that involving the temperature gradient $\partial T/\partial y$, is the amount of radiation passing through the y plane because of emission and scattering by the hairs. The double integral in this term, F_2, increases asymptotically with N_f (as shown in Fig. 2).

If Eq. 9 is evaluated at $\eta = 0.5$, the double integrals in the first two terms are equal. The equation can then be rearranged in the form

$$k_{r-v} = (8\sigma T^3 L F_2)/\pi N_f\,, \tag{13}$$

and

$$q_{r-v}(0.5) = -k_{r-v}\frac{\partial T}{\partial y}\left\{1 + \frac{F_1 N_f}{4F_2}\,\frac{T_a^4 - T_{\infty,r}^4}{(-T^3)L(\partial T/\partial y)}\right\}, \tag{9 a}$$

where the entire right-hand side of both equations is evaluated at $\eta = 0.5$ ($y = L/2$). The contribution of the skin and external diffuse radiation fields is accounted for by the second term in the curly brackets of Eq. 9 *A*. Since the value of this term usually does not exceed 0.1, emission by the hairs is the dominant mechanism of diffuse radiative transfer at the center of fur.

An interesting property of fur can be found by evaluating Eq. 9 at the outer surface of an isothermal fur layer, $\eta = 1.0$. Then the integral in the first term of the Eq. 9 becomes very small, and the integral in the second term is identically equal to $\pi/2$ so that

$$q_{r-v} = \sigma(T^4 - T_{\infty,r}^4), \tag{14}$$

where T is the temperature throughout the fur.

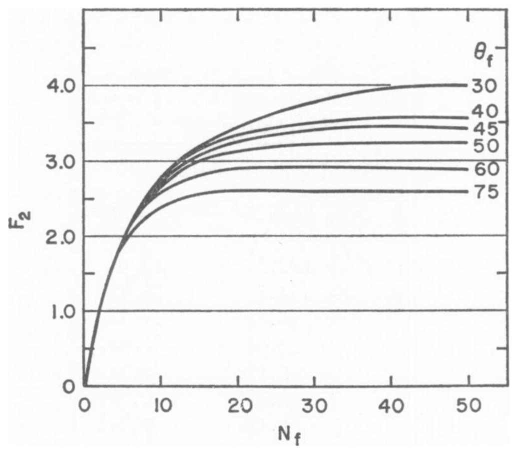

FIGURE 2 F_2 radiation function.

If the emissivity of this fur sample were measured with a pyrometer, Eq. 14 predicts that a value of unity would be obtained and this agrees with the experimental results of Birkebak et al. (1964) and Hammel (1956). This result is independent of the hair emissivity.

The penetration of direct solar radiation into the fur layer can result in significant nonlinearities in the temperature profile. In such a case, the approximation, implicit in Eq. 1,

$$-(\partial T/\partial y)\,|_{y=L/2} \simeq \Delta T/L$$

is not valid.

Several problems immediately arise. We will briefly discuss these in relation to the technique of calculating k_{eff} which is presented subsequently.

It is not a particularly difficult matter to formulate the energy equation, including solar radiation, and to solve numerically for the temperature profile and energy transfer in fur (see Davis, 1972). However, there are nine parameters in this equation and the utility of this method for practical problems is greatly diminished. We thus do not consider this approach to be justified at this time.

In situations where a solution of the governing equation is not attempted, the usual approach involves finding a physically realistic approximation. Experimental data provide the most reliable foundation for such an approximation. Data of the type needed for the problem of the penetration of solar radiation into fur are lacking.

Kovarik (1964) presented a solution of the problem which assumed that k_{eff} was constant throughout the fur layer. Some factors influencing the validity of this assumption will be discussed below.

The direct solar radiation penetrating to a dimensionless distance η from the skin, q_{s-y}, is given by

$$q_{s-y} = \alpha_s' \, S \cos \theta_s \exp(-[1 - \eta]\, N_{f,s} F_s / \cos \theta_s). \tag{15}$$

In this expression α_s' is the fraction of direct solar radiation absorbed by the fur, S is the flux of solar energy passing through a plane whose normal points toward the sun, θ_s is the angle between the normal to the skin and the solar direction, F_s is the function F evaluated at $\theta = \theta_s$, $\phi = \phi_s$ where ϕ_s is the (local) value of the angle ϕ measured from the grain direction to the projection of the solar direction onto the skin. The parameter N_{f_s} is given by

$$N_{f_s} = \beta_s n_f d_f' L/\cos \theta_f . \tag{16}$$

The extinction coefficient in the solar wavelength, β_s, is

$$\beta_s = \alpha_s + \rho_s ,$$

where α_s and ρ_s are, respectively, the absorptivity and reflectivity in the solar wavelengths.

The absorptivity α_s' (whose value is obtained in the usual measurements of solar absorptivity of fur) is not, in general, equal to α_s. Rather, it is a bulk quantity taking into account the entire thickness of the fur and is probably considerably larger than α_s. The product of α_s', S, and $\cos \theta_s$ is the amount of solar energy absorbed within the fur.

If q is the energy flux through the fur in the positive y direction, then from the energy balance in Fig. 3

$$q = q_{s-y} + q_f - q_{s-y}(O), \tag{17}$$

where $q_{s-y}(O)$ is the amount of solar energy which reaches and is absorbed by the

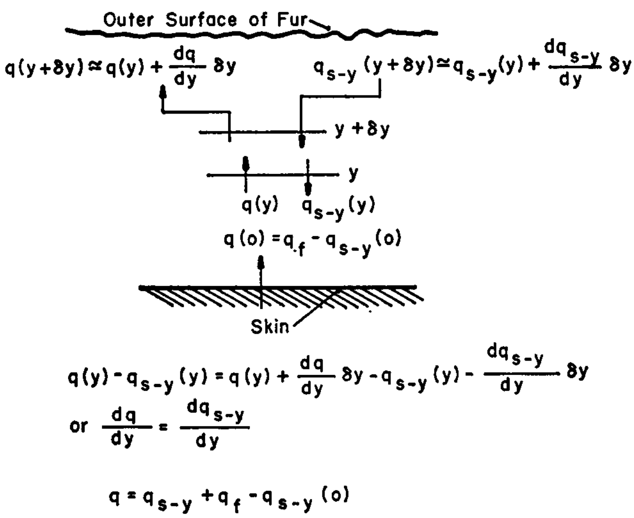

FIGURE 3 Solar energy balance.

skin. The quantity q is also given by

$$q = q_{k-y} + q_{r-y},$$

and this can be cast as

$$q = -k_{eff}(dT/dy). \quad (17\,a)$$

If we assume that k_{eff} is constant (and is calculated in the manner given below) then Eq. 17 a can be substituted into Eq. 17 and the latter solved for the temperature profile and q_f determined as a function of the temperature difference across the fur

$$q_f = (k_{eff}/L)(T_a - T_s) + (1 - [\cos\theta_s/N_{fs}F_s])\,q_{s-y}(O) - ([\alpha_s' S \cos^2\theta_s]/N_{fs}F_s). \quad (18)$$

This is the form of Eq. 1 to be applied in the presence of direct solar radiation.

The term including $q_{s-y}(O)$ is small and, in most applications, should prove to be negligible. The first term on the right-hand side of Eq. 18 is identical to that in Eq. 1 and obviously gives the amount of energy which would be transferred across a fur layer if the temperature profile were linear. The third term takes account of the nonlinearity of the temperature profile induced by the penetration of direct solar radiation. The magnitude of this term is strongly dependent on the value of F_s. Examination of Eq. 8 reveals that F_s is of the order of 0.1 when $\phi_s = 0$, $\theta_s = \theta_f$ (the rays of the sun are parallel to the hairs) and F_s is approximately 0.5–1.0 otherwise.

The ratio of magnitude of the third and first terms in Eq. 18 gives an indication of the error introduced by assuming k_{eff} to be constant. The approximation is certainly valid when this is less than 0.1. More data are necessary for a more precise evaluation of the approximation.

Calculation of Effective Thermal Conductivity

In the absence of direct solar radiation or convective energy transfer within the fur, all of the energy leaving the skin (by nonevaporative mechanisms) must pass through the fur to the surroundings. The sum of the energy conducted, given by Eq. 3, and radiative transfer, given by Eq. 9, must be constant. Thus,

$$q = q_{k-y} + q_{r-y} = \text{constant},$$

and if we write

$$q = q_f = -k_{eff}(\partial T/\partial y), \quad (19)$$

then variations in k_{eff} (e.g., due to fur structure) and $\partial T/\partial y$ are such that their product is constant. We wish to find the mean value of each of these two quantities which, when multiplied together, give the proper value of the energy flux.

In order to accomplish this, we note that measured temperature profiles in fur have been reported by Scholander et al. (1950) and by Hammel (1955). In both cases, the profiles are nearly linear. It follows that the temperature profile in Eq. 19 can be approximated as

$$-(\partial T/\partial y)\,|_{\substack{y=L/2 \\ (\eta=1/2)}} = \Delta T/L,$$

and Eq. 1

$$q_f = -k_{eff}(\eta = \tfrac{1}{2})(\Delta T/L).$$

From Eqs. 3 and 9 *a* it follows that

$$q = \left\{k_v + k_{r-v}\left[1 - \frac{F_1 N_f}{4F_2}\frac{T_a^4 - T_{\infty,r}^4}{T^3 \Delta T}\right]\right\}\frac{\Delta T}{L}, \qquad (19\,a)$$

and so the effective thermal conductivity defined in Eq. 1 is given by

$$k_{eff} = k_v + k_{r-v}\left[1 + \frac{F_1 N_f}{4F_2}\frac{T_a^4 - T_{\infty,r}^4}{T^3 \Delta T}\right]_{\eta=1/2}. \qquad (20)$$

This is the value of effective thermal conductivity used in Eq. 18 when the effects of direct solar radiation are to be included.

A straightforward technique has been devised for finding the value of k_{eff} for a given fur layer. It requires that ρ_{eff}, ρ_f, d_f, d_f', θ_f, L, ϵ_f, k_f, and k_p are known (ϕ_f is taken as equal to 10°). This is a lengthy list of properties. However, the first five are not difficult to measure. The emissivity can be tabulated, at least approximately, as a function of hair diameter. The transverse conductivity, k_p, can be estimated using a flux plot as described previously if ρ_{eff}/ρ_f is known. Thus, the only property difficult to measure or estimate is k_f, the thermal conductivity of hair.

A complete set of property measurements on a piece of tanned deer fur (Davis, 1972) gives the following: $\rho_{eff}/\rho_f = 0.35$, $\epsilon_f = 0.76$, $d_f = d_f' = 0.25$ mm, $L = 26.0$ mm, $\theta_f = 50°$, $k_f = 0.26$ W/mK, $k_p = 0.039$ W/mK, $T = 300$ K, $T_a = 310$ K, $T_{\infty,r} = 290$ K, $\Delta T = 20$ K.

The radiation conductivity is completely determined by ρ_{eff}/ρ_f, d_f, θ_f, and ϵ_f. The dimensionless parameter N_f is calculated from

$$N_f = (4/\pi)(\epsilon_f[\rho_{eff}/\rho_f]L/d_f) = 32.3.$$

From Fig. 2, for this value of N_f and $\theta_f = 50°$,

$$F_2 = 3.25.$$

The radiation function is divided by N_f so the radiation thermal conductivity can be obtained directly from Fig. 4. At a mean temperature in the fur of 300° K and for $L = 26.0$ mm,

$$k_{r-y} = 0.0099$$

(conductivities are in watts per meter-kelvin). From Table I, F_1 is estimated as

$$F_1 \simeq 2 \times 10^{-3},$$

and

$$(N_f F_1/4F_2)([T_a^4 - T_{\infty,r}^4]/T^3 \Delta T) \simeq 0.02.$$

The effect of the skin and external radiation field is small.

The conduction terms also depend on ρ_{eff}/ρ_f and θ_f. From Fig. 5, with $k_g = 0.026$ (conductivity of air at $T = 300$° K),

$$k_1 = \frac{\rho_{eff} k_f}{\rho_f} + \left[1 - \frac{\rho_{eff}}{\rho_f}\right] k_g = 0.108.$$

Then,

$$k_p/k_1 = 0.361,$$

and, from Fig. 6, we have

$$k_y/k_1 = 0.639,$$

and

$$k_y = 0.069.$$

The effective thermal conductivity is then calculated from Eq. 20

$$k_{eff} = 0.079 \text{ W/m-K}.$$

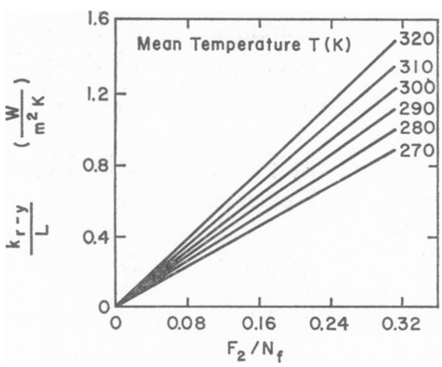

FIGURE 4 Radiative thermal conductivity.

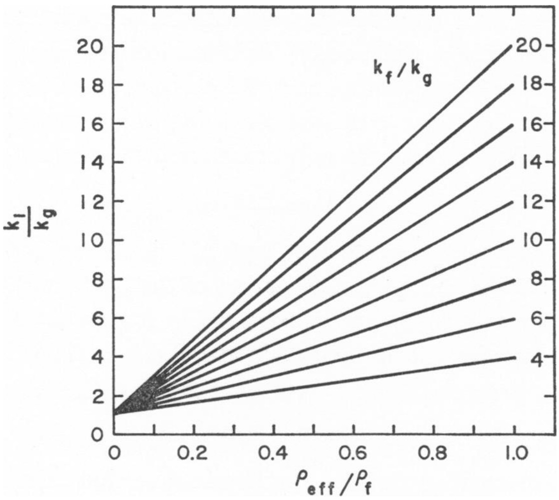

FIGURE 5 Principal direction thermal conductivity.

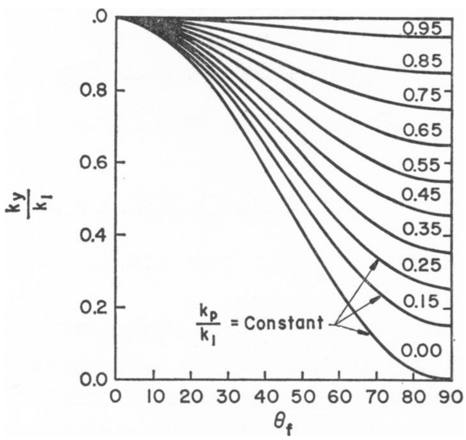

FIGURE 6 Thermal conductivity as a function of fur slant angle.

The effective thermal conductivity of this piece of fur was measured experimentally and found to be 0.091 W/m-K within plus or minus 15% (if the experimental value of k_{eff} is corrected for effects of sample size depth over length of the sample $k_{eff} = 0.084$). The agreement between the experimental and theoretical value is within experimental error.

Convection

Motion of a fluid surrounding a body causes convective energy transfer. There are two types of convection: forced and free. In the former, an externally generated flow field creates the fluid motion. This distorts the temperature field in the fluid in the vicinity of the surface of the body. Fluid motion occurring in free convection is itself a product of buoyancy forces caused by a temperature field. In either case, energy transfer is increased above the value existing in the absence of convection.

The equation for the amount of energy transferred by convection, q_c, is

$$q_c = h\Delta T_c \tag{21}$$

Where h is the heat transfer coefficient and ΔT_c is an appropriate temperature difference (K). Eq. 21 is actually the definition of the heat transfer coefficient.

In dealing with convective energy transfer within fur, we let ΔT_c be equal to ΔT and call the heat transfer coefficient within the fur h_f. Then, in the presence of convection

$$q_f(u_\infty) = h_f\Delta T, \tag{1 a}$$

where $q_f(u_\infty)$ is the energy flux through fur when convection exists. The heat transfer coefficient h_f is a function of the external wind velocity, the temperature field within the fur, body shape, and the fur structure.

The first three of these quantities can be taken into account by use of the dimensionless Nusselt number. This is defined as the ratio of heat transfer in the presence of convection to that in its absence. Thus, dividing Eq. 1 *a* by Eq. 1

$$Nu = q_f(u_\infty)/q_f = h_f L/k_{eff}, \tag{22}$$

or

$$q_f(u_\infty) = Nu\, k_{eff}(\Delta T/L).$$

This quantity was first applied to data from experiments on fur by Birkebak and Cremers (1967).

In order to determine the value of Nu, we have examined (Davis, 1972; Davis and Birkebak, 1973) the combined fluid flow and energy equations and have obtained order of magnitude estimates of the importance of both free convection and forced convection within fur. The results show that free convection is a negligible mode of energy transfer within the fur layer. This is in agreement with results reported by Hammel and Thorington.[2] The examination further indicated that an external flow field can generate forced convection currents within fur whenever the external velocity, u_∞(m/s) exceeds a threshold value that we have called the penetration velocity, u_p(m/s).

The existence of the penetration velocity is illustrated in Fig. 7 using data taken when an air stream impinged in stagnation point flow on a circular sample of caribou fur (Lentz and Hart, 1960).[3] Values for the range of freestream velocities of the heat

[2] Hammel, H. T., and R. Thorington. 1971. Personal communication.

[3] Since this is the only case where the heat transfer coefficient external to the fur can be reliably determined, then of the several sample-free stream orientations for which data are reported by Lentz and Hart, this is the only case where it is actually possible to calculate the value of h_f.

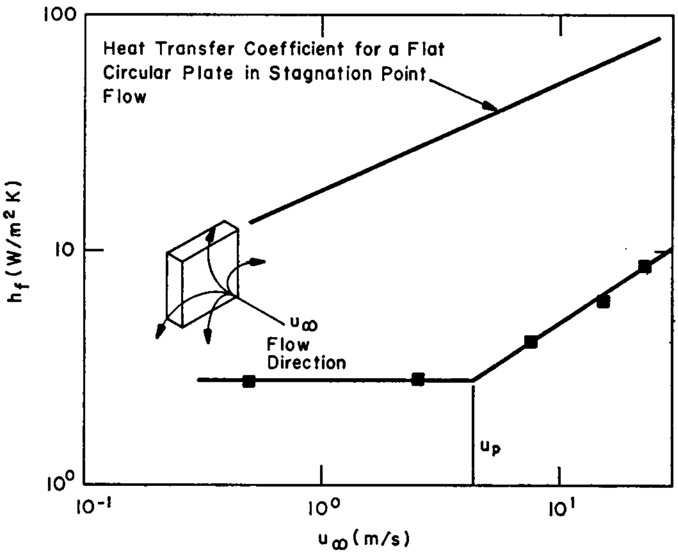

FIGURE 7 Convection heat transfer and penetration velocity in fur.

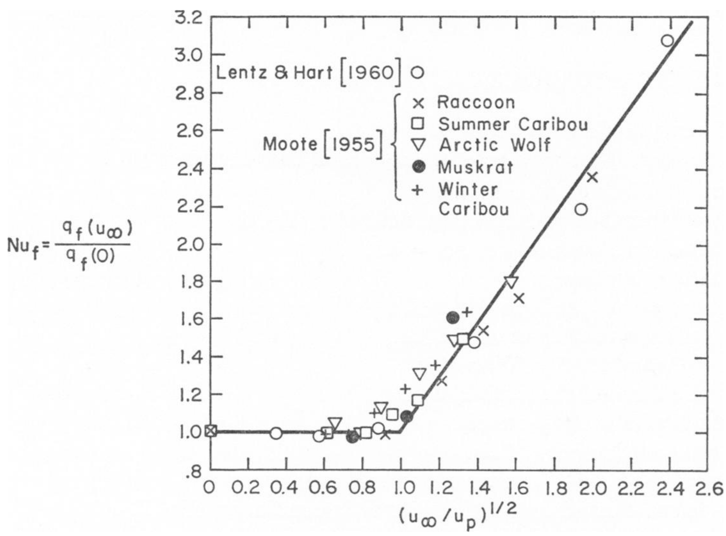

FIGURE 8 Forced convection heat transfer effects in fur.

transfer coefficient over the surface of a bare circular plate in stagnation flow are also presented for comparison. The value of u_∞ at which the rate of increase of h_f with u_∞ becomes approximately equal to that of the heat transfer coefficient of the bare plate is the penetration velocity.

The effects of convective energy transfer within fur can be determined from data correlations of the type shown in Fig. 8, with Nusselt number a function of the dimensionless velocity u_∞/u_p. Additional data from Moote (1955) are included in this figure. A more complete correlation should be obtained from live animal or fur-covered body cast experiments. In these experiments, values of k_{eff}, L, ΔT, q_f, and $q_f(u_\infty)$ should be measured for different freestream airspeeds and orientations of the freestream relative to the body. Values of u_p would be obtained from figures such as Fig. 7 and for each body-freestream orientation there would (potentially) be separate figures such as Fig. 8. These figures would then permit Eq. 22 to be used in calculations of convective energy loss through fur.

CONCLUSIONS

When this study was first begun, there was a paucity of data relating to the thermal conductivity of animal fur and of energy transfer within and from furred surfaces.

In this present study, a theoretical model of fur is developed which allows one to calculate the overall insulation of animal integument as a function of the physical properties of the fur and hairs of the fur layer. The model allows one to take into account the effects of varying ambient air and surrounding temperatures. The end result, the effective thermal conductivity, is used in the thermal modeling equations for heat transfer from animal systems as discussed by Birkebak (1966).

LIST OF SYMBOLS

A	Surface area of an animal (m^2).
d_f	Square root of the average of the square of hair diameters (m).
d_f'	Average of hair diameters (m).
F	Photon mean free path function, defined by Eq. 8.
F_s	The function F with $\theta = \theta_s$, $\phi = \phi_s$.
F_1, F_2	Radiation functions.
h	Heat transfer coefficients (W/m^2K).
h_f	Heat transfer coefficient within fur.
k	Thermal conductivity (W/m-K).
k_{eff}	Effective thermal conductivity of fur.
k_f	Thermal conductivity of hair.
k_g	Thermal conductivity of air.
k_p	Thermal conductivity transverse to hair axes.
k_{r-v}	Equivalent thermal conductivity for diffuse radiation (Eq. 12).
k_k	Equivalent thermal conductivity for conduction (Eq. 4).
k_1	Net thermal conductivity in a direction parallel to the hair axes (Eq. 4 *a*).
L	Fur thickness (m).
N_f	Dimensionless depth of fur, defined by Eq. 10.
N_{f_s}	Dimensionless depth of fur, for direct solar radiation (Eq. 16).
Nu	Nusselt number.
n_f	Number of hairs per unit area of skin (per m^2).
n_f'	Number of hairs per unit area of skin characterized by a particular value of ϕ' ($1/m^2$).
P	Probability that a photon will cross a volume element in fur.
Q	Rate of energy transfer (W).

q Energy flux (W/m^2).
q_c Convective energy flux.
q_f Net energy flux through fur.
$q_f(u_\infty)$ Value of q_f when convection occurs within fur.
q_{k-y} Energy flux through fur by conduction.
q_{r-y} Energy flux through fur by diffuse radiation.
q_{s-y} Energy flux through fur by direct solar radiation.
S Solar radiation flux (W/m^2).
T Temperature (K).
T_a Skin temperature.
$T_{\infty,r}$ Effective temperature of external radiation sink.
u Velocity (m/s).
u_∞ Freestream velocity.
u_p Penetration velocity.
x, y, z Components of Cartesian coordinate system (m).
α Absorptivity of hair.
α_s' Fraction of incident direct solar radiation absorbed by fur.
β Extinction coefficient.
β_s Extinction coefficients in solar wavelengths.
ϵ_f Emissivity of hair.
η Dimensionless coordinate, y/L.
θ Polar coordinate (Fig. 1 *A*).
θ_s Value of θ for the solar direction.
θ_f Value of θ defining the inclination of hairs at a given y-plane.
λ Photon mean free path (Eq. 16 (m)).
ρ Reflectivity of hair.
ρ_{eff} Effective mass density of fur (kg/m^3).
ρ_f Mass density of hair (kg/m^3).
ϕ Aximuthal coordinate (Fig. 1 *A*).
ϕ_s Value of ϕ for the solar direction.
ϕ' Azimuthal angle between the projection of a hair onto the skin and the direction of the grain.
ϕ_f Maximum value of ϕ'.

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